STUDY OF CHANGES IN HYDRODYNAMIC PARAMETERS PATTERNS OF VISCOUS FLUID FLOW IN A FLAT DIFFUSER

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The relevance. Diffusers, either as nozzles or constituent elements, are frequently used in many mechanisms and machines. In this regard, the study of viscous fluid flow in diffusers aims to discover patterns of changes in the flow's hydrodynamic parameters, allowing better understanding of the nature of flow as a function of Reynolds number. Following the results of the analysis of the study, conditions for the proper construction of the mechanism unit, ensuring its reliable and durable operation will be revealed.
The main aim of this study is to determine the velocity profiles in the flat diffuser for a viscous incompressible fluid by integrating the simplified Navier–Stokes differential equations under the established initial and boundary conditions, as well as the bifurcation point's dependence on the opening angle and Reynolds number of the diffuser.
Objects: a flat diffuser in which viscous incompressible fluid moves. At the same time, revealing the patterns of changes of the hydrodynamic parameters of the flow is of defining value when choosing the structural dimension of devices and mechanisms, the main part of which is the flat diffuser.
Methods. To reveal the patterns of changes of the hydrodynamic parameters of the flow in a flat diffuser, the study is based on the fundamental nonlinear differential equations of viscous fluid mechanics, which in a general case are not subject to an exact mathematical solution. For integration in the nonlinear differential equations, due to the smallness, the nonlinear-convective terms are neglected, and the inertial terms are also partially simplified. Such a simplification is justified if the velocities are very small or if the dynamic coefficient of viscosity of the fluid is very large. A method for solving the boundary value problem was developed, and regularities for changing the flow parameters were obtained. According to the derived regularities, graphs of the change in velocity, pressure and shear stresses on the wall of the fixed channel were plotted and the coordinates of the separation point were determined.
Results. Depending on the angle of the diffuser opening and the Reynolds number, a general solution of the approximating Navier–Stokes equations was given. In accordance with the nature of the motion, the boundary conditions of the problem were established and the boundary value problem was stated. A method for integrating a boundary value problem was developed, and regularities for the change in velocities along the length of the diffuser were obtained for a parabolic distribution of velocities in the inlet sections. Graphs of the change in radial velocities along the length and at a fixed value of the opening angle were constructed, a flow pattern and the transition of a single-mode flow to multimode operation were obtained. For a fixed opening angle and Reynolds number, the conditions for flow separation from a fixed wall were derived, where the flow velocity changes the sign.

Key words:
diffuser, velocity profile, pressure distribution, breaking point, viscous fluid, fluid flow.

Introduction

Diffusers are widely used in various mechanisms and machines, either in the form of a nozzle or as an integral part. In this regard, the study of the viscous fluid flow in the diffusers is aimed at identifying patterns of changes in the hydrodynamic parameters, which enables to understand the nature of the flow depending on the Reynolds number. Based on the results of the analysis, the conditions for the correct design of the mechanism assembly, which ensures its reliable and durable operation, will be revealed. Due to the great practical significance, this problem has attracted the attention of many researchers.

The classical problem statement was first formulated by G.B. Jeffery [1] and G. Hamel [2], who proposed the solution of equations of viscous fluid motion in diffusers, taking into account squares of components of velocities and their product multiplication. In further studies, justifications about the effectiveness of this approach were made and solutions were proposed based on the results of

the experimental data. The problems of studying the patterns of changes in the hydrodynamic parameters of the viscous incompressible fluid in flat diffusers were studied by S. Targ [3] and N. Slezkin [4].

However, the solution to such problems was reduced to a system of nonlinear transcendental equations with a complicated integration. Such an approach did not allow making effective calculations for specific parameters of the diffuser. Therefore, the authors proposed more suitable methods for integrating the differential equations of motion in the flat diffuser area. The main point of the studies conducted is that a boundary value problem is formulated and its analytical solution is obtained using Navier–Stokes approximation. Following the solution analysis results, the separation conditions of the flow from the fixed channel are obtained. It should be noted that when deriving these solutions, it was assumed that the velocity on the diffuser axis cannot be equal to zero. However, it became viable to find a class of zero velocity

flows on the diffuser axis when solving this problem. Nevertheless, by analyzing the results of these solutions, it became possible to establish that the flows in the diffuser shall experience bifurcation.

L.D. Akulenko et al. [5–9] studied the generalization of the Jeffrey–Hamel problem solution, obtained conditions for asymmetric stationary flows, and gave one-, two-, and three-mode bifurcation solutions. Conditions for ensuring stationary asymmetric and multi-mode solutions were found for specific intervals of Reynolds numbers and opening angles. Authors of [10] generalized the Jeffrey–Hamel problem solution and deduced conditions of stationary asymmetrical and multi-mode solutions for certain ranges of Reynolds numbers and the diffuser opening angles.

In [11], the author is studying the evolution of the main single-mode stationary flow of the viscous incompressible fluid in the flat diffuser. The Jeffrey–Hamel problem solution is obtained based on the opening angle of the diffuser and Reynolds number. It is established that starting from some critical value of the Reynolds number, the existence of a stationary single-mode flow is impossible. The results of examining several laminar flow regimes in a flat diffuser/confuser with a small opening angle were presented by the authors in [12]. Consequently, patterns of changes in the hydrodynamic parameters of a viscous incompressible fluid was obtained through numerical modeling based on the solution of Navier–Stokes equations. The areas of existence and transitions of flow regimes from stationary-symmetric to stationary-asymmetric and non-stationary ones in the diffuser and confuser, depending on the Reynolds number are found. The values of the Reynolds number, which determine the ranges of the existence of these fluid flow regimes for Newtonian and non-Newtonian fluids are given.

In [13], the author studied the flow regimes in a flat diffuser with a small opening angle, based on the numerical solution of the Navier–Stokes equations for a viscous incompressible fluid. The existence of stationary and non-stationary flow regimes was determined, depending on the Reynolds number. The conditions for the transition of flow regimes in the diffuser from symmetric stationery to asymmetric stationery and then to non-stationary asymmetric ones are obtained. The ranges of the Reynolds numbers for the existence of these regimes are given.

In [14, 15], F. Durst et al. present the results of an experimental study of the flow in a symmetrical expanding channel. Experimental data on flow patterns and velocity profiles in a channel with symmetrical expansion are presented. The authors of [14] experimentally show that the flow in an asymmetric channel with rectilinear expansion can have a stationary and asymmetric nature at low Reynolds numbers.

The fluid flow in diffusers most often occurs in non-stationary and turbulent regimes, therefore, a significant part of the theoretical and experimental studies are devoted to these very regimes in flat diffusers [16, 17]. R.W. Fox and S.J. Kline in [18] give the results of an experimental study of turbulent flows in curvilinear diffusers, which is a continuation and generalization of C. Moore and S.J. Kline's paper [19], where the turbulent flows in diffusers with flat walls were studied.

In [20], the authors obtained the criteria for classifying separations in flat diffusers, as well as diagrams for determining them. Flows in channels and in the diffuser with a small opening angle and at low Reynolds numbers have similar features. Free-jet flows and flows in rapidly expanding channels are margin circumstances of the flow in diffusers. The identification of the condition for violation of the flow symmetry in the flat diffuser and channel, as well as bifurcations in the Jeffrey–Hamel problem were performed in [21].

In [22], the idealized solution of the Jeffrey–Hamel problem for an expanding channel is proposed. Numerical results for a two-dimensional flow in a wedge bounded by two circles are given. The outflow and bifurcation conditions, depending on the Reynolds number are shown. A mathematical model has been created based on studies of changes in the hydrodynamic parameter pattern of a viscous incompressible fluid in the transitional sections of flat pipes, which allowed obtaining results with acceptable accuracy indicating motion dynamics patterns [23]. Water absorption capacity of Irind mine pumice depending on the particle size and absorption time is presented in the paper [24].

Despite a large number of works on the hydrodynamics of a viscous incompressible fluid, new approaches are required to investigate the change in patterns of hydrodynamic flow parameters in flat diffusers. Qualitative characteristic parameters that determine the properties of the motion of a viscous incompressible fluid in the flat diffuser subject to the condition of constant flow rate are the opening angle and the Reynolds number of the diffuser.

Main part

The study of the patterns of change in hydrodynamic parameters of the fluid flow in a flat diffuser with a given velocity profile in its initial section is of great practical interest. Let us consider the problem of viscous fluid flow development in the flat diffuser. The flat diffuser consists of two flat surfaces inclined towards each other at an angle of 2α (Fig. 1), directed along the x axis to infinity. The motion in a flat diffuser will be considered in cylindrical coordinates r, φ starting with the zero point (Fig. 1).

Let's assume that the patterns of radial distribution of the liquid velocity at the inlet section of the diffuser is parabolic, i. e. $\upsilon_r = A(1-\varphi^2)$, at $r=r_0$. The viscous fluid flow in a flat diffuser is considered to be plane-parallel and steady. We will assume, that $\dfrac{\partial \upsilon_r}{\partial r}$ is negligibly small compared to $\dfrac{\partial^2 \upsilon_r}{\partial \varphi^2}$. Assuming also that $\upsilon_\varphi \ll \upsilon_r$ and the derivatives of υ_r to r will be small compared to the derivatives by φ. Discarding the indicated number in the equations of motion [3, 4], we obtain a system of approximate equations:

$$\upsilon_r \frac{\partial \upsilon_r}{\partial r} + \frac{\upsilon_\varphi}{r} \cdot \frac{\partial \upsilon_r}{\partial \varphi} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 \upsilon_r}{\partial \varphi^2}, \qquad (1)$$

$$-\frac{\partial p}{\partial \varphi} + \frac{2\mu}{r}\frac{\partial \upsilon_r}{\partial \varphi} = 0, \qquad (2)$$

$$\frac{\partial \upsilon_r}{\partial r} + \frac{\upsilon_r}{r} + \frac{1}{r}\frac{\partial \upsilon_\varphi}{\partial \varphi} = 0, \qquad (3)$$

where ν and μ are the kinematic and dynamic viscosity coefficients, respectively, υ_r is the fluid velocity in radial directions (Fig. 1). All the notations are well known [3, 4].
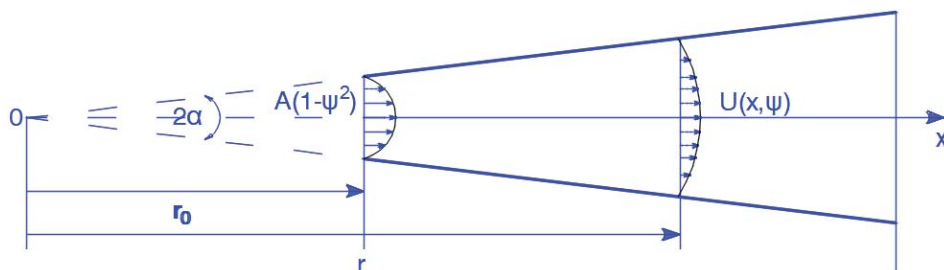

Fig. 1. *Calculation scheme of a flat diffuser*
Рис. 1. *Расчетная схема плоского диффузора*

Having in mind, that υ_φ is a negligibly small value, we can take $\upsilon_\varphi = 0$, and the value υ_r can be replaced for a given section with an average flow rate U:

$$U = \frac{Q}{2\alpha r}.$$

Here Q is the fluid consumption through each section. The condition of fluid flow constancy is derived as:

$$Q = \int_{-\alpha}^{\alpha} \upsilon_r \cdot r d\varphi. \qquad (4)$$

Substituting the regularity of the initial distribution of the radial velocity in (4) we obtain:

$$Q = \int_{-\alpha}^{\alpha} \upsilon_r \cdot r_0 d\varphi = 2Ar_0\alpha\left(1 - \frac{\alpha^2}{3}\right).$$

The average flow rate in the initial section of the diffuser will be:

$$U_0 = A\left(1 - \frac{\alpha^2}{3}\right).$$

Then equation (1) can be written as:

$$U\frac{\partial \upsilon_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 \upsilon_r}{\partial \varphi^2} \qquad .(5)$$

Equations (2), (3) and (5) constitute a system of approximate equations of fluid flow to identify patterns of changes in the hydrodynamic parameters of a viscous fluid in a flat diffuser. The characteristic flow rate U is taken to be the rate included in the Reynolds number formula [3]:

$$\mathrm{Re} = \frac{Ur\alpha}{\nu}.$$

Based on the condition in this mode, maintaining a constant value of the number Re leads to dependence of U on r:

$$U = \frac{U_0 r_0}{r} = \frac{A\left(1 - \frac{\alpha^2}{3}\right)r_0}{r}.$$

To obtain universal solutions to the problem, we introduce dimensionless variables u, v, ψ, x, $\overline{p}$, assuming:

$$u = \frac{\upsilon_r}{U_0}, \ V = \frac{\upsilon_\varphi}{\alpha U_0}, \ \psi = \frac{\varphi}{\alpha}, \ x = \frac{r}{r_0}, \ \overline{p} = \frac{p}{\rho U_0^2}, \quad (6)$$

where $U_0 = \dfrac{Q}{2\alpha r_0}$ is the average rate in the inlet sections of the diffuser.

Equations (2), (3) and (5) for the new variables (6) (will take the form:

$$\begin{cases} \dfrac{1}{x}\dfrac{\partial u}{\partial x} = -\dfrac{\partial \overline{p}}{\partial x} + \dfrac{\nu}{r_0 U_0 \alpha^2}\dfrac{\partial^2 u}{x^2 \partial \psi^2} \\[2mm] -\dfrac{\partial \overline{p}}{\partial \psi} + \dfrac{2\nu}{U_0 r_0 x}\dfrac{\partial u}{\partial \psi} = 0 \\[2mm] \dfrac{\partial u}{\partial x} + \dfrac{u}{x} + \dfrac{1}{x}\dfrac{\partial V}{\partial \psi} = 0 \end{cases} \qquad (7)$$

From the second equation of system (10), after integration over the angle ψ, the equation is defined:

$$\overline{P}(x,\psi) = \frac{2\nu}{U_0 r_0}\frac{u}{x} + C(x), \qquad (8)$$

where $C(x)$ is an unknown function of x.

Differentiating (8) by x, we get:

$$\frac{\partial \overline{P}(x,\psi)}{\partial x} = 2\frac{\nu}{U_0 r_0}\frac{\partial}{\partial x}\left(\frac{u}{x}\right) + \frac{dC(x)}{dx}. \qquad (9)$$

If we substitute the expression (6) into the right side of the first equation (9) and discard the member containing the value $\dfrac{\partial}{\partial x}\left(\dfrac{u}{x}\right)$ as a member of a lower order, the system of equations (7) is finally transformed to the form:

$$\begin{cases} \dfrac{1}{x}\dfrac{\partial u}{\partial x} = \dfrac{a^2}{x^2}\dfrac{\partial^2 u}{\partial \psi^2} - \dfrac{dC(x)}{dx} \\[2mm] \dfrac{\partial u}{\partial x} + \dfrac{u}{x} + \dfrac{1}{x}\dfrac{dV}{d\psi} = 0 \end{cases} \qquad , \qquad (10)$$

where designated $a^2 = \dfrac{\nu}{r_0\alpha^2 U_0}$.

To integrate the system of equations (10), the boundary conditions are established:

1) conditions for adhesion between the liquid and the wall's surface:

$$x>0, \ \psi=\pm1, \ u=0, \ V=0, \qquad (11)$$

2) conditions for the symmetry of the velocity profile along the flow section:

$$\psi = 0, \quad \frac{\partial u}{\partial \psi} = 0,$$

3) conditions for the radial velocity distribution over the input arc section are presented by some function $f(\psi)$, i. e. when

$$x=0, \ -1\le\psi\le+1, \ u(1,\psi)=A(1-\psi^2). \qquad (12)$$

From the solution of the equation of system (10) we get the form of the sum, i. e.

$$u(x,\psi) = \sum_{k=1}^{\infty} F_k(x)\cos\lambda_k\psi + \sum_{k=1}^{\infty} W_k(x)\cos\lambda_k\psi. \qquad (13)$$

where $F_k(x)$ and $W_k(x)$ are continuous functions to be determined. The value $F_k(1)$ can be calculated from the boundary condition (12), so we will have:

$$A(1-\psi^2) = \sum_{k=1}^{\infty} F_k(1)\cos\lambda_k\psi. \qquad (14)$$

By multiplying both parts of equation (14) by $\cos\lambda_n\psi$ and integrating in the interval $(-1; 1)$ we get:

$$A\int_{-1}^{1}(1-\psi^2)\cos\lambda_n\psi d\psi =$$

$$= \sum_{k=1}^{\infty} F_k(1)\int_{-1}^{1}\cos\lambda_k\psi\cos\lambda_n\psi d\psi. \qquad (15)$$

Using the orthogonality property of the function $\{\cos\lambda_n\psi\}$, i. e.

$$\int_{-1}^{1}\cos\lambda_k\psi\cos\lambda_n\psi d\psi = \begin{cases} 0, & \lambda_k \neq \lambda_n \\ 1, & \lambda_k = \lambda_n \end{cases},$$

and calculating the integral value on the left side of the equation (15):

$$A\int_{-1}^{1}(1-\psi^2)\cos\lambda_n\psi d\psi = \frac{4(-1)^{n+1}}{\lambda_k^3}$$

we can get

$$F_k(1) = C_k = \frac{4(-1)^{k+1}}{\lambda_k^3},$$

where $\lambda_k = (2k-1)\dfrac{\pi}{2}$ are the eigenvalues, the roots of the eigenfunctions: $\cos\lambda_k\psi=0$.

The first equation of system (10), taking into account (13), will be rewritten in the form:

$$\sum_{k=1}^{\infty} F_k'(x)\cos\lambda_k\psi + \sum_{k=1}^{\infty} W_k'(x)\cos\lambda_k\psi =$$

$$= -\frac{a^2}{x}\sum_{k=1}^{\infty}\lambda_k^2 F_k(x)\cos\lambda_k\psi -$$

$$-\frac{a^2}{x}\sum_{k=1}^{\infty}\lambda_k^2 W_k(x)\cos\lambda_k\psi - xC'(x). \qquad (16)$$

We can expand the function $C'(x)$ in a series as eigenfunctions:

$$C'(x) = \sum_{k=1}^{\infty} A_k(x)\cos\lambda_k\psi,$$

where

$$A_k(x) = \int_{0}^{1}\sum_{k=1}^{\infty} C'(x)\cos\lambda_k\psi d\psi = \frac{(-1)^{k+1}}{\lambda_k}C'(x). \qquad (17)$$

From equations (16), taking into account (17), we can find:

$$F'(x) + \frac{\beta_k^2}{x}F(x) = -\left(W_k'(x) + \frac{\beta_k^2}{x}W_k(x) + xA_k(x)\right),$$

$$(\beta_k^2 = a^2\lambda_k^2).$$

We choose an arbitrary function $W_k(x)$ in a way, to satisfy the conditions:

$$W_k'(x) + \frac{\beta_k^2}{x}W_k(x) = -xA_k(x), \qquad (18)$$

$$F'(x) + \frac{\beta_k^2}{x}F(x) = 0. \qquad (19)$$

Solving equation (18), we get:

$$W_k(x) = x^{-\beta_k^2}B_k(x), \qquad (20)$$

where $B_k(x)$ is the arbitrary constant to be determined.

Solving equations (20) and together, we obtain: $B'_k(x) = -x^{\beta_k^2+1}A_k(x)$, whence

$$B_k(x) = -\frac{(-1)^{\kappa+1}}{\lambda_\kappa}\int_{1}^{b}t^{\beta_k^2+1}C'(t)dt. \qquad (21)$$

Solving equations (19), we get:

$$F_k(x) = C_k x^{-\beta_k^2}. \qquad (22)$$

Substituting the value of the functions $B_k(x)$ and $A_k(x)$ from equations (21) and (22) into (13) we will have:

$$u(x,\psi) = \sum_{k=1}^{\infty} AC_k x^{-\beta_k^2}\cos\lambda_k\psi -$$

$$-x\sum_{k=1}^{\infty}\frac{\partial}{\partial x}\left[\frac{(-1)^{k+1}}{\lambda_k}x^{-\beta_k^2}\int_{1}^{x}t^{\beta_k^2+1}C'(t)dt\right]\cos\lambda_k\psi =$$

$$= \sum_{k=1}^{\infty}\left[AC_k - \frac{(-1)^{k+1}}{\lambda_k}\int_{1}^{x}t^{\beta_k^2+1}C'(t)dt\right]x^{-\beta_k^2}\cos\lambda_k\psi. \qquad (23)$$

Substituting the value $u(x,\psi)$ from (23) into the second equation of system (10) for determining the $V(x,\psi)$ function, we get the equation:

$$-\frac{\partial V(x,\psi)}{\partial \psi} =$$

$$= x\sum_{k=1}^{\infty}\frac{\partial}{\partial x}\left[AC_k - \frac{(-1)^{k+1}}{\lambda_k}\int_{1}^{x}t^{\beta_k^2+1}C'(t)dt\right]x^{-\beta_k^2}\cos\lambda_k\psi +$$

$$+\sum_{k=1}^{\infty}\left[AC_k - \frac{(-1)^{k+1}}{\lambda_k}\int_{1}^{x}t^{\beta_k^2+1}C'(t)dt\right]x^{-\beta_k^2}\cos\lambda_k\psi.$$

After simple transformations, the last equation will take the form:

$$\frac{\partial V(x,\psi)}{\partial\psi} =$$

$$= \sum_{k=1}^{\infty}(\beta_k^2 - 1)x^{-\beta_k^2}\left[\begin{array}{c}\dfrac{4A(-1)^{k+1}}{\lambda_k^3} - \dfrac{(-1)^{k+1}}{\lambda_k}\times\\ \times\displaystyle\int_1^x t^{\beta_k^2+1}C'(t)dt - \dfrac{x^{\beta_k^2+2}}{\beta_k^2-1}C'(t)\end{array}\right]\cos\lambda_k\psi.$$

We choose an arbitrary function $C(x)$ so that to satisfy the condition:

$$\int_1^x t^{\beta_k^2+1}C'(t)dt - \frac{x^{\beta_k^2+2}}{\beta_\kappa^2-1}C'(x)=0 . \qquad (24)$$

Denoting $t^{\beta_k^2+1}C'(t)=y(x)$ the last equation is transformed into the form:

$$\int_1^x y(t)dt - \frac{x}{\beta_\kappa^2-1}y(x)= 0.$$

Differentiating it with x, we get:

$$xy'(x)-\left(\beta_\kappa^2-2\right)y(x)= 0 \text{ or } \frac{d}{dx}\left(\ln y(x)\right)=\frac{\left(\beta_\kappa^2-2\right)}{x},$$
from where $y(x)=x^{\beta_k^2-2}$.

Taking in mind the value $y(x)$ for $C'(x)$ the equation is defined as:

$$C'(x)= x^{-3},$$

where

$$C(x)=C(1)+\frac{x^2-1}{2x^2}. \qquad (25)$$

Taking into account conditions (24), the equation for determining the function $v(x,\psi)$ will take the form:

$$\frac{\partial V(x,\psi)}{\partial\psi} = \sum_{k=1}^{\infty}(\beta_k^2 - 1)x^{-\beta_k^2}\frac{4A(-1)^{k+1}}{\lambda_k^3}\cos\lambda_k\psi.$$

Integrating this equation in the interval $(-1;\psi)$ and keeping in mind the boundary conditions (11), it can be expressed as:

$$V(x,\psi)=\sum_{k=1}^{\infty}x^{-\beta_k^2}\frac{4A(-1)^{k+1}(\beta_k^2-1)}{\lambda_k^4}[\sin\lambda_k\psi + (-1)^{k+1}],$$
$$\text{when } -1\le\psi\le 0, \qquad (26)$$

$$V(x,\psi)=\sum_{k=1}^{\infty}x^{-\beta_k^2}\frac{4A(-1)^{k+1}(\beta_k^2-1)}{\lambda_k^4}[(-1)^k - \sin k\psi],$$
$$\text{when } 0\le\psi\le 1. \qquad (27)$$

Substituting the eigenvalues into equations (26) and (27), we finally get:

$$V(x,\psi)=-\frac{64A}{\pi^4}\sum_{k=1}^{\infty}x^{-\beta_k^2}\frac{(\beta_k^2-1)}{(2k-1)}[(-1)^{k+1}\sin(k-0.5)\pi\psi+1],$$
$$\text{when } -1\le\psi\le 0,$$

$$V(x,\psi)=-\frac{64A}{\pi^4}\sum_{k=1}^{\infty}x^{-\beta_k^2}\frac{(\beta_k^2-1)}{(2k-1)}1-[(-1)^{k+1}\sin(k-0.5)\pi\psi],$$
$$\text{when } 0\le\psi\le 1.$$

For the function $u(x,\psi)$, taking into account the values C_k and $C'(x)$, the formula will be expressed as:

$$u(x,\psi)=\sum_{k=1}^{\infty}\left[\frac{4A(-1)^{k+1}}{\lambda_k^3} - \frac{(-1)^{k+1}}{\lambda_k}\frac{x^{\beta_k^2-1}-1}{\beta_k^2-1}\right]x^{-\beta_k^2}\cos\lambda_k\psi.$$

Taking into account the value of the eigenvalues $\lambda_k = (2k-1)\frac{\pi}{2}$, we finally get:

$$u(x,\psi)=\sum_{k=1}^{\infty}\left[\frac{4Ax^{-\beta_k^2}}{\pi^3(k-0.5)^3} - \frac{x^{-1}-x^{-\beta_k^2}}{\pi(k-0.5)(\beta_k^2-1)}\right]\times$$
$$\times(-1)^{k+1}\cos[(k-0.5)\pi]\psi. \qquad (28)$$

The resulting solution satisfies all boundary conditions.

We can calculate the patterns of pressure change from (7), (25) and (28):

$$\bar{p}(x,\psi)=$$

$$= \frac{2\nu}{U_0 r_0}\sum_{k=1}^{\infty}\left[\frac{4Ax^{-\beta_k^2}}{\pi^3(k-0.5)^3} - \frac{x^{-1}-x^{-\beta_k^2}}{\pi(k-0.5)(\beta_k^2-1)}\right]\times$$
$$\times(-1)^{k+1}\cos[(k-0.5)\pi]\psi + C(1)+\frac{x^2-1}{2x^2}. \qquad (29)$$

The value of constant integration is calculated from the initial condition, at $x = 1$, $\bar{p}(x,\psi)= \bar{p}_0$, and we get:

$$C(1)= \bar{p}_0(1,0)-\frac{2\nu}{U_0 r_0}\sum_{k=1}^{\infty}\frac{4A(-1)^{k+1}}{\lambda_k^3} . \qquad (30)$$

As

$$\sum_{k=1}^{\infty}\frac{4A(-1)^{k+1}}{\lambda_k^3} = \frac{32A}{\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^3} = A,$$

From (30) we can get:

$$C(1)= \bar{p}_0(1,0)-\frac{2\nu A}{U_0 r_0} . \qquad (31)$$

Substituting the value $C(1)$ from (31) into (29) we finally get the pattern of pressure change along the length of the flat diffuser:

$$\bar{p}(x,\psi) - \bar{p}(1.0)=$$

$$= \frac{2\nu}{U_0 r_0}\left\{A - \sum_{k=1}^{\infty}\left[\frac{4x^{-\beta_k^2}}{\pi^3(k-0.5)^3} - \frac{x^{-1}-x^{-\beta_k^2}}{\pi(k-0.5)(\beta_k^2-1)}\right]\times\right.$$
$$\left.\times(-1)^{k+1}\cos[(k-0.5)\pi]\psi\right\} +$$
$$+\frac{x^2-1}{2x^2}.$$

On the wall of the stationary channel, due to the velocity gradient and viscosity, shear stresses are formed, which is determined by the formula [4]:

$$\tau_{r,\varphi} = \mu\left(\frac{1}{r}\cdot\frac{\partial\upsilon_r}{\partial\varphi} + \frac{\partial\upsilon_\varphi}{\partial r} - \frac{\upsilon_\varphi}{r}\right). \qquad (32)$$

In view of the negligible transverse velocity component υ_φ, compared to the derivative of υ_r tangential stress on the angle of φ, the shear stress on the diffuser (32) wall will be determined by the formula:

$$\tau = \frac{\mu}{r}\cdot\left(\frac{\partial\upsilon_r}{\partial\varphi}\right)_{\varphi=\alpha}$$

or dimensionless form:

$$\bar{\tau} = \frac{\nu}{\alpha U_0 r_0} \cdot \frac{1}{x} \cdot \frac{\partial u}{\partial \psi}\bigg|_{\psi=1} . \qquad (33)$$

Substituting the expression for the radial velocity (28) into (33) we obtain the formula for determining the dimensionless shear stress on the diffuser wall:

$$\bar{\tau} = -\frac{\nu}{\alpha U_0 r_0} \cdot \sum_{k=1}^{\infty} \left[\frac{x^{-2} - x^{-\beta_k^2-1}}{(\beta_k^2-1)} - \frac{4Ax^{-\beta_k^2-1}}{\pi^2(k-0.5)^2} \right] \times$$
$$\times (-1)^{k+1} Sin[(k-0.5)\pi]\psi \big|_{\psi=1} . \qquad (34)$$

Based on the expression obtained, we get the place of flow separation from the diffuser wall in accordance with the condition, that separation occurs at the place, where the shear stresses become zero:

$$(\bar{\tau})_{\psi=1} = 0 .$$

From the last equation, taking into account (34), for determination of the unknown parameter, we obtain:

$$\frac{x^{-2} - x^{-\beta_k^2-1}}{(\beta_k^2-1)} - \frac{4Ax^{-\beta_k^2-1}}{\pi^2(k-0.5)^2} = 0 .$$

Discussion of the results

Based on the solutions obtained, we study the nature of the flow features in a flat diffuser. From the obtained equations for the distribution of velocities $u(x,\psi)$ and $V(x,\psi)$ it follows that for $x \to \infty$, $u(\infty,\psi) \to 0$ and $V(x,\psi) \to 0$. These conditions are fully consistent with the condition of constant flow.

The graphs were plotted in order to visualize the patterns of changes in the radial velocity $u(x,\psi)$ along the transverse section and along the length of a flat diffuser, as well as the shear stress on the wall of a fixed channel, depending on the opening angle $\alpha=20°$ 10°, 5° and the Reynolds number Re=20, 40, 60, 80, 100, 110. Fig. 2–5 show the indicated graphs for cases $\alpha=20°$ and 5° at Re=60 and Re=70.

Numerical calculations were carried out at a constant value: $A=0.7$.

The separation point is a special point for the shear stress function (32) where

$$\beta_k^2 - 1 = 0 \text{ or } \lambda_k^2 = \alpha \text{Re} . \qquad (35)$$

Under condition (35), the value $\bar{\tau}$ is undefined. The indeterminacy is found according to the L'Hopital's rule [6]. As a result, it turns out:

$$\sum_{k=1}^{\infty} \lim_{\beta_k^2 \to 1} \left[\frac{x^{-2} - x^{-\beta_k^2-1}}{(\beta_k^2-1)} - \frac{2.8\alpha \text{Re } x^{-\beta_k^2-1}}{\beta_k^2} \right] (-1)^{k+1} = 0 ,$$

whence follows the condition

$$x^{-1}\left(\frac{2.8}{\alpha \text{Re}} - \ln x \right) = 0 \text{ or } x = \exp\frac{2.8}{\alpha \text{Re}} . \qquad (36)$$
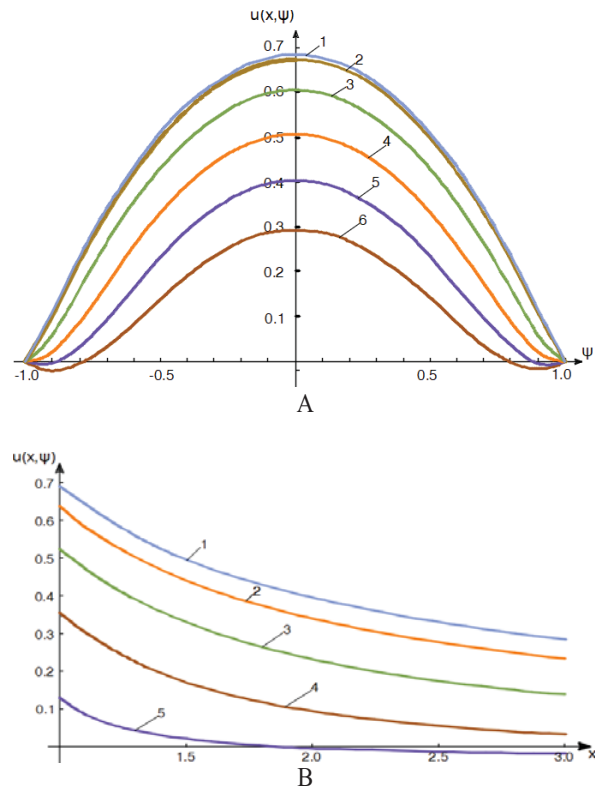

Fig. 2. *Graphs of changes in the radial velocity u(x,ψ) in a flat diffuser at an opening angle α=20° and the Reynolds number Re=60. A) along the cross-section at 1. x=1.03, 2. x=1.05, 3. x=1.2, 4. x=1.5, 5. x=2.0, 6. x=3.0; B) lengthwise at 1. ψ=0.1, 2. ψ=0.3, 3. ψ=0.51, 4. ψ=0.7, 5. ψ=0.9*

Рис. 2. *Графики изменения радиальной скорости u(x,ψ) в плоском диффузоре при угле раствора α=20° и числе Рейнольдса Re=60: A) по поперечному сечению при 1. x=1.03, 2. x=1.05, 3. x=1.2, 4. x=1.5, 5. x=2.0, 6. x=3.0; B) по длине при 1. ψ=0.1, 2. ψ=0.3, 3. ψ=0.51, 4. ψ=0.7, 5. ψ=0.9*

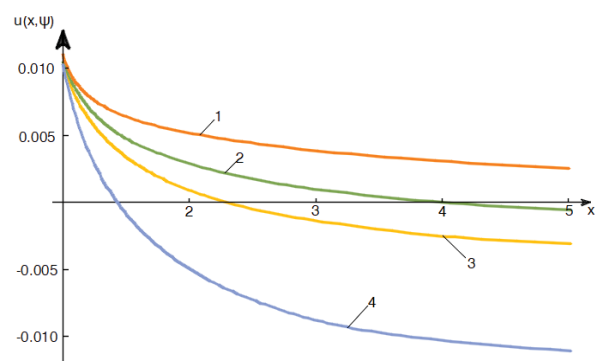**Fig. 3.** *Graph of changes in shear stresses in a flat diffuser at α=20° and Reynolds numbers 1. Re=10, 2. Re=20, 3. Re=30, 4. Re=40*

Рис. 3. *График изменения касательных напряжений в плоском диффузоре при α=20° и числах Рейнольдса 1. Re=10, 2. Re=20, 3. Re=30, 4. Re=40*

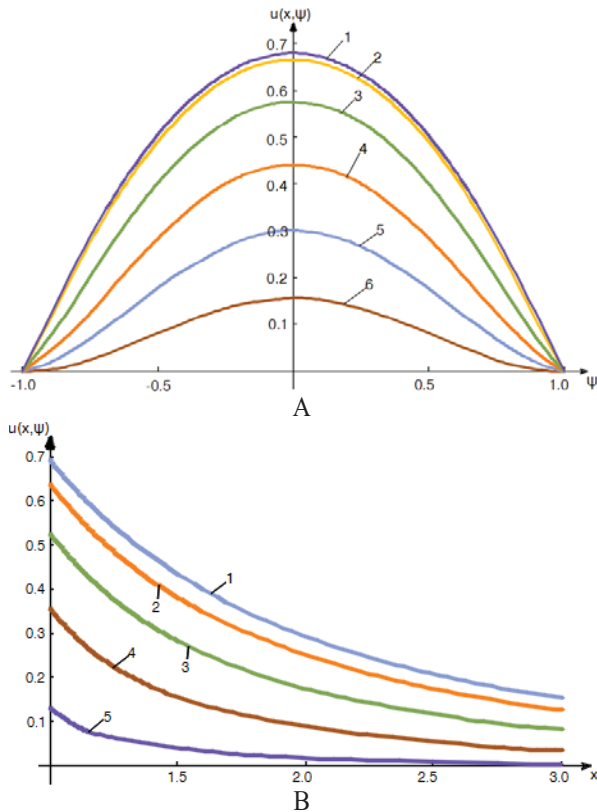**Fig. 4.** *Graphs of changes in the radial velocity u(x, ψ) in a flat diffuser at an opening angle α=5° and the Reynolds number Re=60: A) along the cross-section at 1. x=1.03, 2. x=1.05, 3. x=1.2, 4. x=1.5, 5. x=2.0, 6. x=3.0; B) lengthwise at 1. ψ=0.1, 2. ψ=0.3, 3. ψ=0.51, 4. ψ=0.7, 5. ψ=0.9*

Рис. 4. *Графики изменения радиальной скорости u(x, ψ) в плоском диффузоре при угле раствора α=5° и числе Рейнольдса Re=60: A) по поперечному сечению при 1. x=1.03, 2. x=1.05, 3. x=1.2, 4. x=1.5, 5. x=2.0, 6. x=3.0; B) по длине при 1. ψ=0.1, 2. ψ=0.3, 3. ψ=0.51, 4. ψ=0.7, 5. ψ=0.9*

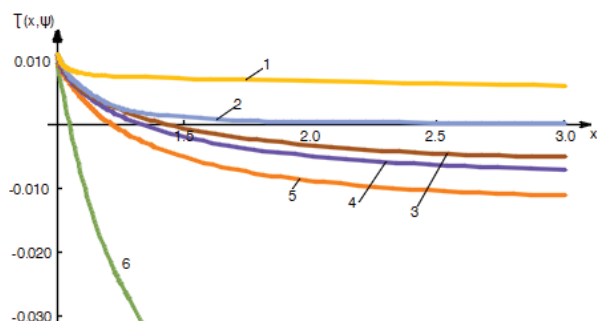**Fig. 5.** *Graph of changes in shear stresses in a flat diffuser at α=5° and Reynolds numbers 1. Re=10, 2. Re=20, 3. Re=30, 4. Re=40, 5. Re=50, 6. Re=60*

Рис. 5. *График изменения касательных напряжений в плоском диффузоре при α=5° и числах Рейнольдса 1. Re=10, 2. Re=20, 3. Re=30, 4. Re=40, 5. Re=50, 6. Re=60*

For a visual representation of the place of separation, a graph of the function *x* from αRe (36) is shown in Fig. 6.
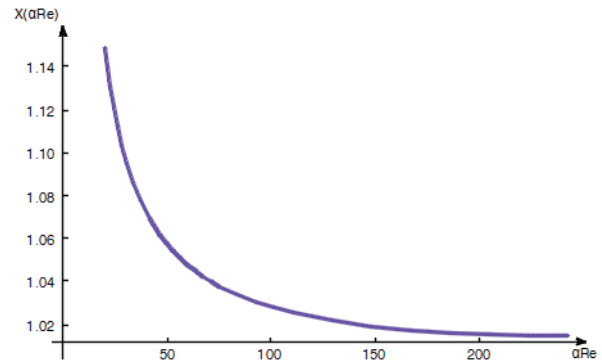

Fig. 6. *Function graph x from αRe*
Рис. 6. *График функци x от αRe*

The analysis of the numerical calculation results are presented in the form of graphs (Fig. 2, B; 3; 4, B; 5; 6) showed that the coordinates of the separation points were determined depending on the opening angle and Reynolds number. The viscous liquid flow to the separating point is considered stationary and strictly flat-parallel and, according to the results of calculations, the hydrodynamic parameters are strictly symmetrical (Fig. 2, 4). The nature of the flow is disturbed after the separation point, and the obtained solutions do not provide accurate results. However, they can be used for qualitative analysis. At the separation points, the sign of the shear stress and radial velocity change. However, they can be used for qualitative analysis. At the separation points, the sign of the shear stress and radial velocity change.

It can be seen from the graph that the coordinates of the separation point exactly match the data determined from the graphs. In addition, it can be seen that the conditions for continuous flow in a flat diffuser at small opening angles are possible at significantly higher Reynolds numbers. As a result, the single-mode stationary flow is sharply reduced (Fig. 2, 3), as a result of which the stationary regime is disturbed. Multimode flow starts, accompanied by various pulsation processes and unstable operation of the diffuser, where the obtained solutions are invalid. The main goal of diffuser design is to ensure a steady mode of operation, which can be achieved by choosing the optimum dimensions.

Conclusion

Based on the study results, the following conclusions were formulated:

- the features of the viscous fluid flow in flat diffusers are determined;
- the method for solving a boundary value problem was developed, and formulas for calculating radial and transverse velocities, shear stresses on the wall of a fixed channel, and pressure along the length of the diffuser were obtained;
- the graphs of changes in the flow's hydrodynamic parameters and shear stress on the channel wall were designed;
- the coordinate of the flow separation point was determined using the opening angle and the Reynolds number, which is the main parameter of the diffuser.

The obtained solutions of the approximating Navier–Stokes equations for identifying the patterns of changes in hydrodynamic parameters in a flat channel, make it possible to identify the main point of ongoing processes and determine the bifurcation point coordinates from the diffuser opening angle and the Reynolds number. The critical values of the Reynolds number are also determined in case the regime transfers from symmetrical to asymmetrical. The pressure and shear stress regularities of variations on the fixed channel's wall are found along the length of the flat diffuser, and the coordinates of the separation point are defined, as it is shown on the graphs.

Based on the results of the studies obtained, it is possible to correctly design a flat diffuser, to choose the opening angle and its length according to the condition of continuous movement. Flat diffusers are the main part in many technological equipment for exploration, mining, transportation and processing of geo-resources.

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ИССЛЕДОВАНИЕ ИЗМЕНЕНИЯ ЗАКОНОМЕРНОСТЕЙ ГИДРОДИНАМИЧЕСКИХ ПАРАМЕТРОВ ТЕЧЕНИЯ ВЯЗКОЙ ЖИДКОСТИ В ПЛОСКОМ ДИФФУЗОРЕ

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Актуальность. *В различных механизмах и машинах широко используются диффузоры, либо в виде насадки, либо в качестве составной части. В связи с этим исследование движения вязкой жидкости в диффузорах направлено на выявление закономерностей изменения гидродинамических параметров потока, что позволит лучше понять характер движения в зависимости от числа Рейнольдса. По результатам анализов исследования выявятся условия по правильному конструированию узла механизма, обеспечивающего его надежную и долговечную работу.*
Целью *настоящей работы является исследование закономерностей изменения гидродинамических параметров вязкой несжимаемой жидкости в плоском диффузоре и определение параметров потока в фиксированном сечении.*
Объекты: *плоский диффузор, в котором двигается вязкая несжимаемая жидкость. При этом выявление закономерностей изменения гидродинамических параметров потока имеет определяющее значение при выборе конструктивных размеров аппаратов и механизмов, основной частью которых является плоский диффузор.*
Методы. *В основу исследования по выявлению закономерностей изменения гидродинамических парамеров потока в плоском диффузоре заложены фундаментальные нелинейные дифференциальные уравнения механики вязкой жидкости, которые в общем случае не поддаются точному математическому решению. С целью интегрирования в нелинейных дифференциальных уравнениях, ввиду малости, отброшены нелинейно-конвективные члены, а также упрощены инерционные члены. Такое упрощение оправдано, если скорости весьма малы или если динамический коэффициент вязкости жидкости весьма велик. Разработан метод решения краевой задачи, получены закономерности изменения параметров потока. По выведенным закономерностям построены графики изменения скорости, давления и касательных напряжений на стенке неподвижного канала и определены координаты точки отрыва.*
Результаты. *В зависимости от угла раствора диффузора и числа Рейнольдса дано общее решение аппроксимирующих уравнений Навье–Стокса. В соответствии с характером движения установлены граничные условия задачи и сформулирована краевая задача. Разработан метод интегрирования краевой задачи, получены закономерности изменения скоростей по длине диффузора при параболическом распределении скоростей во входном сечении. Построены графики изменения радиальных скоростей по длине и при фиксированном значении угла раствора, получена картина течения и переход одномодового течения к многомодовым режимам. При фиксированном значении угла раствора и числа Рейнольдса выведены условия отрыва потока от неподвижной стенки, при которых скорость потока меняет знак.*

Ключевые слова:
диффузор, профиль скорости, распределение давления, предел устойчивости, вязкая жидкость, течение жидкости.

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